Dimensional Analysis

Introduction

Dimensional analysis is a way to convert a quantity given in one unit to an equal quantity of another unit by lining up all the known values and multiplying. It is sometimes called factor-labeling. The best way to start a factor-labeling problem is by using what you already know. In some cases you may use more steps than a classmate to find the same answer, but it doesn't matter. Use what you know, even if the problem goes all the way across the page!

In a dimensional analysis problem, start with your given value and unit and then work toward your desired unit by writing equal values side by side. Remember you want to cancel each of the intermediate units. To cancel a unit on the top part of the problem, you have to get the unit on the bottom. Likewise, to cancel a unit that appears on the bottom part of the problem, you have to write it in on the top.

Once you have the problem written out, multiply across the top and bottom and then divide the top by the bottom.

Example: 3 years = ? seconds

Step 1: Start with the value and unit you are given. There may or may not be a number on the bottom.

$$\left[\frac{3 \text{ years}}{\quad}\right]$$

Step 2: Start writing in all the values you know, making sure you can cancel top and bottom. Since you have years on top right now, you need to put years on the bottom in the next segment. Keep going, canceling units as you go, until you end up with the unit you want (in this case seconds) on the top.

$$\left[\frac{3 \text{ years}}{\quad}\right] \times \left[\frac{365 \text{ days}}{1 \text{ year}}\right] \times \left[\frac{24 \text{ hours}}{1 \text{ day}}\right] \times \left[\frac{60 \text{ minutes}}{1 \text{ hour}}\right] \times \left[\frac{60 \text{ seconds}}{1 \text{ minute}}\right]$$

Step 3: Multiply all the values across the top. Write in scientific notation if it's a large number. Write units on your answer.

$$3 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^7 \text{ seconds}$$

Step 4: Multiply all the values across the bottom. Write in scientific notation if it's a large number. Write units on your answer if there are any. In this case everything was cancelled so there are no units.

$$1 \times 1 \times 1 \times 1 = 1$$

Step 5: Divide the top number by the bottom number. Remember to include units.

$$9.46 \times 10^7 \text{ seconds} / 1 = 9.46 \times 10^7 \text{ seconds}$$

Step 6: Review your answer to see if it makes sense. 9.46×10^7 is a really big number. Does it make sense for there to be a lot of seconds in three years? YES! If you had gotten a tiny number, then you would need to go back and check for mistakes.

In lots of APES problems, you will need to convert both the top and bottom unit. Don't panic! Just convert the top one first and then the bottom.

Example: 50 miles per hour = ? feet per second

Step 1: Start with the value and units you are given. In this case there is a unit on top and on bottom.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right]$$

Step 2: Convert miles to feet first.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right] \times \left[\frac{5280 \text{ feet}}{1 \text{ mile}}\right]$$

Step 3: Continue the problem by converting hours to seconds.

$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right] \times \left[\frac{5280 \text{ feet}}{1 \text{ mile}}\right] \times \left[\frac{1 \text{ hour}}{60 \text{ minutes}}\right] \times \left[\frac{1 \text{ minute}}{60 \text{ seconds}}\right]$$

Step 4: Multiply across the top and bottom. Divide the top by the bottom. Be sure to include units on each step. Use scientific notation for large numbers.

$$50 \times 5280 \text{ feet} \times 1 \times 1 = 264000 \text{ feet}$$
$$1 \times 1 \times 60 \times 60 \text{ seconds} = 3600 \text{ seconds}$$
$$264000 \text{ feet} / 3600 \text{ seconds} = 73.33 \text{ feet/second}$$

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet. Use scientific notation when appropriate.

Conversions:
1 square mile = 640 acres
1 hectare (Ha) = 2.47 acres
1 kw-hr = 3,413 BTUs
1 barrel of oil = 159 liters
1 metric ton = 1000 kg

1. 134 miles = ? inches
2. 8.9×10^5 tons = ? ounces
3. 1.35 kilometers per second = ? miles per hour
4. A city that uses ten billion BTUs of energy each month is using how many kilowatt-hours of energy?
5. A 340 million square mile forest is how many hectares?
6. If one barrel of crude oil provides six million BTUs of energy, how many BTUs of energy will one liter of crude oil provide?
7. Fifty eight thousand kilograms of solid waste is equivalent to how many metric tons?